Implementation of ROSETTA in SysML v2 and Underlying Mathematical Formalisms

Mathsig Presentation to SE DSIG

21st of March 2023

Prof C.E. Dickerson

Loughborough University

mathsig/23-03-01

Context

- The Mathematical Formalism DSIG is working on foundational formalisms that underly MBSE and can be expressed via OMG model-based standards [1].
- The Relational Oriented Systems Engineering Technology Trade-off & Analysis framework (ROSETTA) is a mathematical foundation for understanding MBSE from a relational viewpoint. Constraints were difficult to model in SysML v1.
- In June 2018, UPR 1.0 [2] was adopted (the UML Profile for ROSETTA).
- Over the past five years, further research and commercialisation has continued.
- More recently the UPR Revision Task Force (RTF) has followed the progress of SysML v2 with an expectation that ROSETTA concepts can be expressed in SysML.

This is a report of the initial findings on implementing ROSETTA in SysML v2.

Implementing ROSETTA and UPR in SysML v2

Initial findings from an elementary radar design problem

- SysML v2 Language Capabilities [3]
  - Requirements → Constraint Definition → metric expressions → UPR
  - Behavior
  - Structure → Definition Elements → ROSETTA
  - Analysis → 'Solver' e.g., Maple
  - Verification
  - View & Viewpoint → Graphical or Tabular → ROSETTA

Note: need stable SysML v2 API

- The matrix framework for ROSETTA can represent graphs and tables.
- Maple has been used as a a solver for Analysis Capabilities (Jet Engine).
- Binary relations between Definition Elements → ROSETTA

ROSETTA: Relational Oriented Systems Engineering Technology Trade-off & Analysis framework
UPR: UML Profile for ROSETTA [2]

Topics of the Presentation

- The *elementary radar design problem* from an engineering viewpoint
  - An air traffic control radar 'triggering' the transponder on an aircraft
  - Multi-objective multi-attribute analysis
- Mathematical interpretation of an ISO/IEC/IEEE 15288:2015 concept
  - Requirements transformation as a mathematical transformation
  - Constraint expressions, requirements compliance and consistency in ROSETTA
- Associating Engineering Elements with SysML v2 Definition Elements
- Future topics: formalisation of Use Case Diagrams with KerML

Demonstration of how foundational formalisms that underly MBSE can be expressed via systems engineering standards

An elementary radar design problem [4]*

- A local airport is seeking to upgrade its air traffic control service:
  - The controlled airspace will be from ground level to 10 kft for a 30 nmi radius.
  - Radar tracking of aircraft will be introduced.
  - Participating aircraft will be required to use a transponder.
  - The transponder replies to radar illumination with a 100 W encoded signal.**
  - Transponders and radars must be purchased from regulated suppliers.
- Electromagnetic radiation from the radar is also regulated.
  - A physically secure hazard zone must be agreed upon by the regulators.
  - The power density of the radar signal must be $\leq$ 50W/m^2 outside the zone [5].

*This is adapted from the tutorial chapters (5-7) of the textbook [4].
**The signal is much stronger than any possible radar echo. The encoding in the signal is used for aircraft identification.

Technical details of the radar and design trade-offs

- System elements and design characteristics of the radar elements*:
  - Transmitter: P_t = transmitter power (W)
  - Antenna:

    G_t = gain of the transmitting antenna (unitless)

    A_e = effective aperture (electronic area) of the antenna (m^2)

  Note: $G = 4\pi A_e / \lambda^2$, where λ is the wavelength of the radio frequency energy (m).
  - Signal Processor: M = minimum detectable signal (sensitivity) measured in mW**
- Power-aperture product $P_t A_e$ is a key metric but emitted power is $P_t G_t$.

*The system terminology is from Section 6.4.5 Design Definition in [6]. Characteristics are attributes in SysML.

**milliwatts.

Power-aperture product vs. power density trade-offs*

Multi-objective multi-attribute analysis

- The elementary radar design problem is to:
  - *Maximise* power-aperture product: $z = P_t A_e$ (W-m^2); define $f(P_t, A_e) = P_t A_e$
  - *Subject* to the power density constraint: $P_t G_t/4\pi d^2 \leq 50$ W/m^2 at the perimeter d
  - The units are different and must be normalised before making comparisons.
    Note: $G = 4\pi A_e/\lambda^2$ implies $P_t G_t/4\pi d^2 = P_t (4\pi A_e)/4\pi d^2\lambda^2 = P_t A_e/d^2\lambda^2$
  - The comparison is made at the perimeter d. The wavelength λ is fixed at 0.3m.**
  - Define $c = d\lambda$ and specify $d = (1/0.3)$ m = (3.3…) m, then $c^2 = d^2\lambda^2 = 1\text{m}^4$.
- The problem is then to maximise $z = f(P_t, A_e)$ subject to $P_t A_e \leq 50$ W-m^2.
- The 'normalisation constant' c scales the constraint to 50 (W/m^2)c^2.

*The propagation spreading loss of a radar signal at distance d is $1/4\pi d^2$. The energy spreads over a sphere of area $4\pi d^2$.

**Regulations fix the transponder frequency at 1 GHz which corresponds to a wavelength of 0.3m [(300x10^6 m/s)/(10^9/s)].
Other choices for the perimeter radius d could be made but d = (1/0.3) m is simple and reasonable.

Topics of the Presentation

- The *elementary radar design problem* from an engineering viewpoint
  - An air traffic control radar 'triggering' the transponder on an aircraft
  - Multi-objective multi-attribute analysis
- **Mathematical interpretation of an ISO/IEC/IEEE 15288:2015 concept**
  - **Requirements transformation as a mathematical transformation**
  - **Constraint expressions, requirements compliance and consistency in ROSETTA**
- Associating Engineering Elements with SysML v2 Definition Elements
- Future topics: formalisation of Use Case Diagrams with KerML

The elementary radar design problem is sufficient to demonstrate of how foundational formalisms that underly MBSE can be used to express the concept of requirements transformation in ISO/IEC/IEE 15288:2015.

Transformation of requirements into technical views
ISO/IEC/IEEE 15288:2015 System Requirements Definition [6]

6.4.3.1 *The purpose of the System Requirements Definition process is to **transform*** the stakeholder, user-oriented **view of desired capabilities into a technical view** of a solution …*

- In the elementary radar design problem, the user-oriented *view of EMF safety* was represented by a constraint (power density $\leq$ 17 dBW/sm) which was ***transformed*** *into a mathematical model of the solution* (i.e., a technical view)

  $P + A_e \leq 17$ (dBWsm) at the safety perimeter (3.3m)

- The concept of radar system power-aperture product was scaled to power density at 3.3 m to convert the EMF safety constraint (dBW/sm) into units of dBWsm that can be compared with power-aperture (the system characteristic).
- Transformation: $f(P, A_e) = P + A_e$ constrained by scaling 17 dBW/sm to dBWsm

**This is a demonstration of how mathematical formalisms that underly MBSE can be expressed in terms of systems engineering standards.*

Transformation of requirements into a technical view

Solution set for EMF safety requirement

The power-aperture product is linearised using decibels (dB):

$z = P_t A_e$ (W-m^2) → $z = x_1 + x_2$ (dBWsm)

Emitted power:

$P = P_t G$ (W) note: gain is unitless

$G = 4\pi A_e / \lambda^2$ implies $P = P_t A_e / d^2\lambda^2$.

The EMF safety requirement is

$R \leq 50$ W/m^2 (17 dBW/sm)

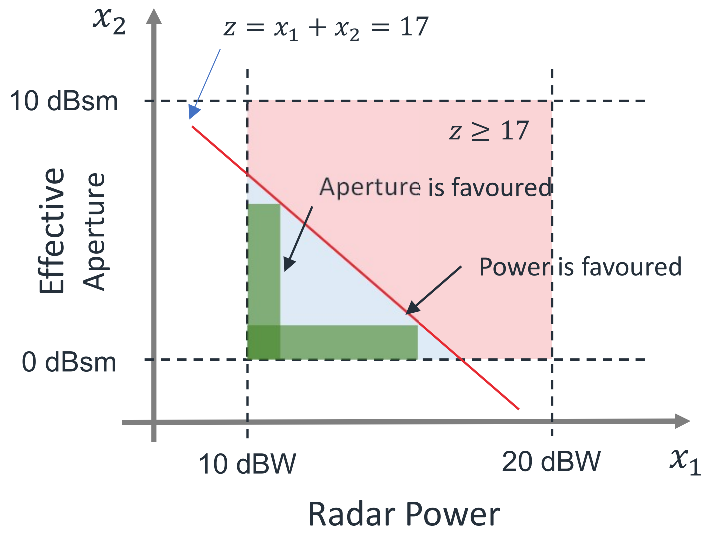
This is applied at the perimeter, which is a distance d from the radar (antenna). The units for *z*, P, and R are all different and not comparable. Radar power density at the perimeter is,

$P/4\pi d^2 = P_t A_e / d^2\lambda^2$ (W/m^2) is comparable to R (W/m^2)

The conversion* $c = d\lambda$ and $R^* = Rc^2$ permits the comparison

$P \leq R^*$ (W-m^2) or in dB, $z = x_1 + x_2 \leq 17$ (dBWsm).

For c =1 m^2, $R^* = Rc^2 = R$ (W/m^2)(1 m^4) = R (Wsm).

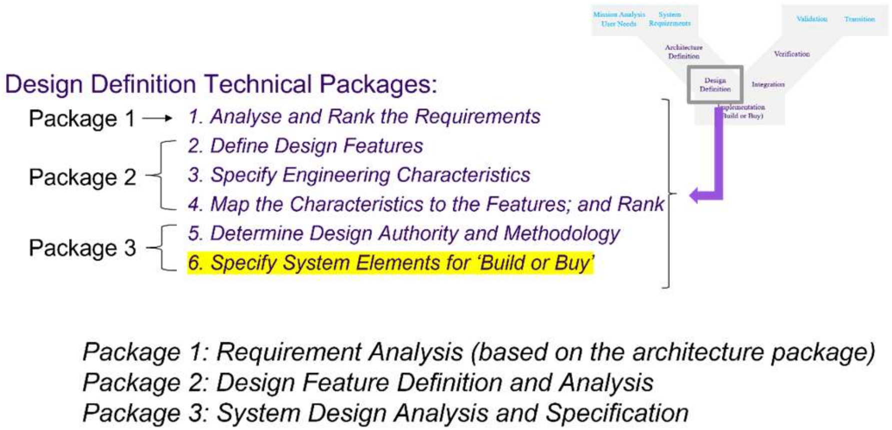


Representation in UPR 1.0

*The conversion is the result of a *design decision* (hazard perimeter) with implications on radar installation → civil engineering <<requirements>>.

System Analysis and Element Specification*

The technical view enables Design Definition (in ISO/IEC/IEEE 15288:2015)



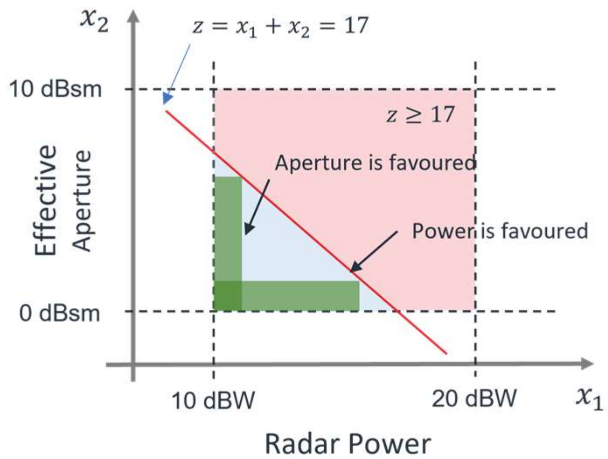
Preconditions include

Transformation of stakeholder requirements to system solution:

Power aperture = $z = f(x_1, x_2) = x_1 + x_2 \leq 17$

$z \in [0, 17] \rightarrow (x_1, x_2)$ is a solution

The constraint normalised to dBWsm implies a *design decision* on perimeter.



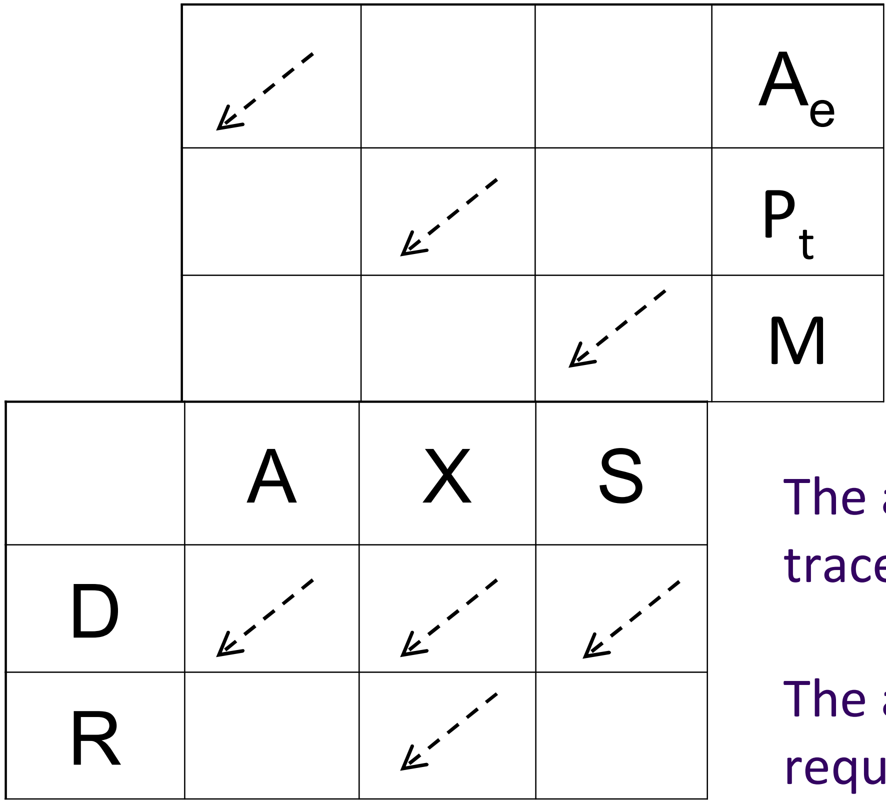
Outcomes include

- Decisions from trade-offs
- System element and interface specifications
- Associated models
- Supporting analysis
- Traceability to the architecture

*Adapted from the Loughborough University WS66 System Design MSc module.

Representation in ROSETTA: Multi-Model Traceability

Chains of Binary Relationships in the radar models



(A_e, A) (A, D): (A_e, D)

(P_t, X) (X, D): (P_t, D)

(P_t, X) (X, R): (P_t, R)

(M, S) (S, D): (M, D)

The attributes A_e, P_t, and M are traceable to requirement D.

The attribute P_t is traceable to requirement R.

A_e, P_t, M: Radar design characteristics [Relations taken from the Radar Equation]
A, X, S: Antenna, Transmitter, Signal Processor (system elements)
D, R: Detection, Radiation requirements

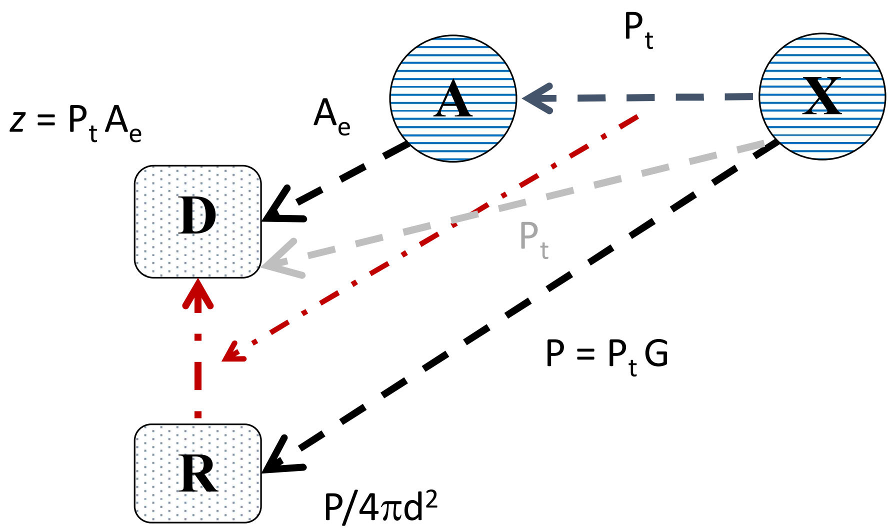
The Algebra of Relational Transformation:

Graphical Representation of relations between radar models

A new relation between requirements is identified.

D: Detection Requirement

R: Radiation Requirement



- Relational transformation is based on
  - Mathematical morphism
  - Algebraic graph theory
- Given the (relation) edge (X, A), and
  - The (relations) edges(X, R) and (A, D); then
  - (X, A) *transforms* into (R, D)
- *(R, D): implied relation between R and D. This is the conversion relation when applied at the safety perimeter.*

A: Antenna

X: Transmitter

RD: *Power density → Conversion relation → Power-aperture product*

Topics of the Presentation

- The *elementary radar design problem* from an engineering viewpoint
  - An air traffic control radar ‘triggering’ the transponder on an aircraft
  - Multi-objective multi-attribute analysis
- Mathematical interpretation of an ISO/IEC/IEEE 15288:2015 concept
  - Requirements transformation as a mathematical transformation
  - Constraint expressions, requirements compliance and consistency in ROSETTA
- **Associating Engineering Elements with SysML v2 Definition Elements**
- Future topics: formalisation of Use Case Diagrams with KerML

Foundational formalisms that underly MBSE: mathematical and relational transformation can express requirements transformation. How can SysML v2 Definition Elements be used to do this?

Relevant SysML v2 diagrams (textual)

```
part def Radar{
        attribute powerAperture:> powerAperture = radarPower + effectiveAperature;
        part def Transmitter{
                attribute radarPower:> dBW;
        }
        part def Antenna{
                attribute effectiveAperture:> dBsm;
        }
}
requirement def SafetyRequirement {
        doc Power density AL shall be less than or equal to 17dBW/sm
        attribute safetyPerimeter:> ISQ::length;
        attribute powerDensity:> dBW/sm;
        require constraint {powerDensity <=17dBW/sm}
}
```

Requirements and Design Definition in ROSETTA

Formalisms for System Design that can be expressed in SysML v2

SysML v2 Definition Elements enable basic requirements and design definition:

✓ X: Design Space — $X = X_1 \times X_2$ = Power – Aperture space

$S = [p_1, p_2] \times [a_1, a_2]$ an 'orthotope' in X (Solution)

✓ Z: Objective Space — Z = Power – Aperture Product space

$z = f(x_1, x_2) = x_1 + x_2$ (dBWsm) (Knowledge mapping)

✓ R: Requirement — $R = [r_1, r_2]$ e.g., [0, 50] (W/sm)* (Power density Constraint)

What is the Definition Element or Usage for the *requirements transformation*?

The requirements transformation included a conversion constant $c \geq 0$ dBsm (1 m^2).

Power density → Power-aperture product: $r \rightarrow r\,c^2 = r$ *for* $c = 1\ m^2$; $z = x_1 + x_2 \leq r$ *(dBWsm) for* $c = 1\ m^2$

*[0, 17] (dBW/sm); the conversion process includes a *design decision* about the safety perimeter ($d = c/\lambda$).

Initial findings on implementing ROSETTA in SysML v2

It is clear that UPR 1.0 can be implemented in SysML v2 using requirements constraints.

*Basic concepts of engineering design can be implemented but it is not clear how system **relations between relations** can be.*

Topics of the Presentation

- The *elementary radar design problem* from an engineering viewpoint
  - An air traffic control radar ‘triggering’ the transponder on an aircraft
  - Multi-objective multi-attribute analysis
- Mathematical interpretation of an ISO/IEC/IEEE 15288:2015 concept
  - Requirements transformation as a mathematical transformation
  - Constraint expressions, requirements compliance and consistency in ROSETTA
- Associating Engineering Elements with SysML v2 Definition Elements
- **Future topics: formalisation of Use Case Diagrams with KerML**

The initial findings demonstrate how basic foundational formalisms that underly MBSE can be expressed via SysML v2; but relational?

Findings on radar design insights into KerML are also available.

Discussion and Feedback

- Discussion with the SE DSIG
- Further discussion with Mathsig members and others:
  Mathsig meeting Wednesday afternoon 13:00 – 14:00
- A short Mathsig paper will be written and submitted to the SE DSIG.

References

[1] OMG Mathsig Homepage, https://www.omg.org/maths/

[2] OMG, UPR: UML Profile for ROSETTA, v1.0, 2019, https://www.omg.org/spec/UPR

[3] SysML Version 2.0 Release 2023-02, https://github.com/Systems-Modeling/SysML-v2-Release/blob/master/doc/2-OMG_Systems_Modeling_Language.pdf

[4] Dickerson, C., & Ji, S. (2021). *Essential architecture and principles of systems engineering*. CRC Press.

[5] The UK Secretary of State, “The Control of Electromagnetic Fields at Work Regulations”, Health and Safety Regulation 2016 No. 588.

[6] *Systems and Software Engineering – System Life Cycle Processes*, ISO/IEC/IEEE 15288:2015.